TENTH WORLD CONGRES ON THE THEORY OF
MACHINE AND MECHANISMS
Oulu, Finland, June 20 – 24, 1999

EVALUATION OF THE 64 "INSENSITIVITY" POSITIONS FOR A 6-RKS HUNT-TYPE PARALLEL MANIPULATOR

Zabalza I.*, Pintor J. M.*, Ros J.* and Jimenez J. M.§
* Mechanical, Energy and Materials Department, Public University of Navarra, Spain, [izavi, txus, jros]@upna.es
§ Sports Training Technologies, S.L., San Sebastian, Spain, jmjimenez@sportstt.com

Keywords: robot, parallel manipulator, sensitivity analysis, singularity.

Abstract. In this paper, the insensitivity positions (IP) of a 6-RKS-like parallel manipulator are analysed. This parallel robot topology was introduced by Hunt. This paper highlights that the IP can be split into two different categories depending on whether they have "positive" kinematic characteristics or not. The first group of IP is called "positive insensitivity positions" (PIPs) and the second is called "negative insensitivity positions" (NIPs). In this work it is shown that the 6-RKS parallel manipulator has 64 PIPs. An iterative method has been implemented for the calculation of all the PIPs. This method is verified in a practical example.

1. Introduction

Insensitivity positions (IP) are singular positions of mechanical linkages in which the mechanical system suddenly suffers a change in the number of degrees of freedom it has. Examples of this phenomenon can be found both in planar and three-dimensional systems. In the planar case, the slider-crank mechanism and some configurations of the four-bar linkage are typical examples. The slider-crank mechanism shows an IP when the crank and the coupler are aligned; analogously, the four bar mechanism shows an IP when the input element is a crank and it is aligned with the coupler. In those singular positions, the position of the output element is not affected by a small perturbation in the position of the input element and the velocity of the output element is always negligible independently of the velocity of the input element. In addition, the external loads applied on the output element are supported by the reaction forces at the kinematic joints and hence the torque in the input element vanishes. These singular positions of planar linkages are often used in automation to obtain positions in which the mechanism stays at rest with a very high precision. Due to its "positive" characteristics, these singular positions are called "positive insensitivity positions" (PIPs). In opposition, the singular positions in which the output element gains a degree of freedom are called "negative insensitivity positions" (NIPs) because in those configurations, the output link can not be controlled by the actuators located in the input elements.

In this paper, the PIPs of a 6-RKS parallel manipulator are analysed. An iterative procedure for determining the 64 PIPs is presented and its effectiveness is demonstrated through a practical example.

2. Robot topology and notation

The 6-RKS parallel manipulator was introduced by Hunt [Hunt 1983] and is composed of two triangular platforms one of them fixed to the ground. On the fixed platform there are six rotating actuators (R) located on the edges of the triangle. These actuators are the input elements on which the motors are located. Each actuator is linked to the moving platform through a rod. In each rod, one tip is linked to an actuator through a universal joint (K) and the other tip is connected to the moving platform by means of a spherical joint (S), as depicted in figure 1.

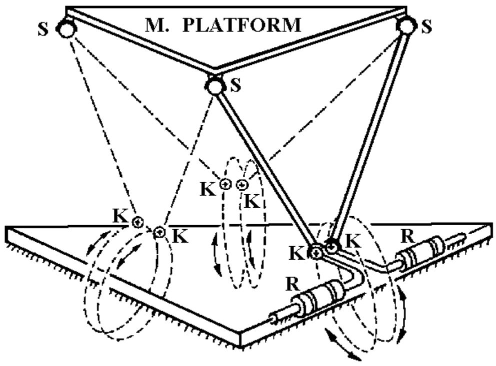

Fig. 1. The 6-RKS parallel manipulator introduced by Hunt.

The notation used to describe the topology of this parallel manipulator is summarised in the following points and shown in figure 2.

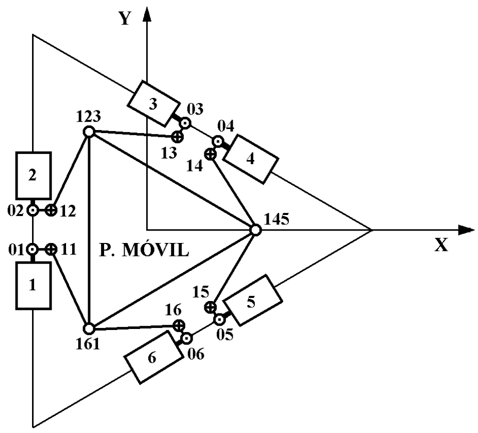

Fig. 2. Notation used to describe the parallel manipulator.

- *0i*: centre of the rotation axis of the *i-th* actuator.
- *1i*: centre of the universal joint between the *i-th* actuator and the *i-th* rod. The segment defined by points *0i* and *1i* is perpendicular to the actuator axis.
- R_i: length between centres *0i* and *1i*.
- L_i: length of the *i-th* rod.
- *1ik*: centre of the spherical joint between rods *i-th* and *k-th*. These centres coincide with the vertices of the moving platform.
- A_{pq}: length of the edge of the moving platform that links the *p-th* and *q-th* rods.

3. 6-RKS Parallel Manipulator "Positive insensitivitity positions" (PIPs)

Several authors have focused in the study of the "insensitivity positions" of parallel manipulators. [Stewart 1965] proposed a parallel manipulator and mentioned that the robot has two positions of instability that are NIPs. In [Hunt 1983] two types of "special configurations" are described, which are called "stationary singularity" and "uncertainty singularity", respectively. The first one corresponds to a PIP in which the platform does not move when small perturbations are introduced to the input elements; the second one is a NIP in which the platform can move without any displacement of the input elements. Other authors have also studied this problem ([Merlet 1988], [Gosselin et al. 1990], [Zlatanov et al. 1995], [Benea 1996], [Takeda et al. 1997]), giving different names to those singular positions. All those authors develop methods to eliminate the singular positions from the manipulator workspace and no one tries to take advantage of their "positive" characteristics.

Analysing the topology of the 6-RKS parallel manipulator, it can be considered that this mechanism is composed of six spatial slider-crank mechanisms. By analogy with the planar slider-crank mechanism, it is clear that this robot may have PIPs in all those configurations in which each rod and its corresponding actuator stand in the same plane. This condition was mentioned by [Freudenstein et al. 1969] in the analysis of the singular configurations of a spatial four-bar linkage. [Nombrail 1993] referred to these singular positions in the study of robots controlled by rotational actuators and more recently, [Benea 1996] studied these singular positions in the analysis of a prototype of the 6-RKS parallel manipulator.

In this paper, it is demonstrated that the mentioned positions are PIPs. In those positions the position of the moving platform is not affected by small perturbations of the position of the input elements and, analogously, its velocity vanishes for any value of the velocity of the input elements. Moreover, the reactions transmitted by the rods do not produce any torque in the actuators axis and, hence, the motor efforts are zero. Consequently, those PIPs can be used to design positions that can be reached by the moving platform with high accuracy.

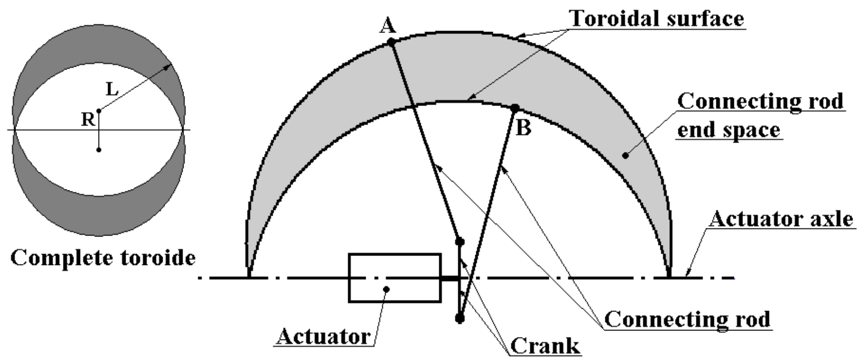

Fig. 3 Toroidal surfaces for a PIP.

Let consider a configuration in which the moving platform is located over the platform fixed to the ground. Each pair actuator-rod shows two PIPs. The first one (A) corresponds to the position in which the rod is almost aligned with the actuator while the second (B) corresponds to the position in which those elements nearly overlap. In each PIP, the tip of the rod will stand on the surface of a toroidal region, as depicted in figure 3. The 6-RKS parallel manipulator has six pairs actuator-rod having two PIPs each, so the manipulator may show up to 64 different PIPs. This result was discussed by [Benea 1996].

4. Calculation of the 64 PIPs for the 6-RKS Parallel Manipulator

To model the 6-RKS parallel manipulator, a set of *natural coordinates* [García de Jalón et al. 1994] is used. *Natural coordinates* is a set of coordinates that define the position of all the elements of the mechanical system with respect to an inertial reference frame using the Cartesian coordinates of some points and the Cartesian components of some unit vectors, usually located at the kinematic joints. An inertial reference frame is considered on the fixed platform, as shown in figure 2, taking the Z axis from the ground towards the moving platform. Giving the lengths R_i and L_i, the set of constraint equations that define the kinematics of the robot can be written as follows.

- One constant length condition for each actuator

$$(x_{11}-x_{01})^2+(y_{11}-y_{01})^2+(z_{11}-z_{01})^2-R_1^2=0 \qquad \text{(1 to 6)}$$

- One constant length condition for each rod

$$(x_{161}-x_{11})^2+(y_{161}-y_{11})^2+(z_{161}-z_{11})^2-L_1^2=0 \qquad \text{(7 to 12)}$$

- One constant length condition for each edge of the moving platform

$$(x_{161}-x_{123})^2+(y_{161}-y_{123})^2+(z_{161}-z_{123})^2-A_{12}^2=0 \qquad \text{(13 to 15)}$$

- One perpendicular condition for each actuator

$$(x_{11}-x_{01})(x_{02}-x_{01})+(y_{11}-y_{01})(y_{02}-y_{01})+(z_{11}-z_{01})(z_{02}-z_{01})=0 \qquad \text{(16 to 21)}$$

- One restriction to force each pair actuator-rod to stand in the same plane

$$\begin{aligned}&(x_{02}-x_{01})(y_{11}-y_{01})(z_{161}-z_{11})-(x_{02}-x_{01})(z_{11}-z_{01})(y_{161}-y_{11})+\\&(y_{02}-y_{01})(z_{11}-z_{01})(x_{161}-x_{11})-(y_{02}-y_{01})(x_{11}-x_{01})(z_{161}-z_{11})+\\&(z_{02}-z_{01})(x_{11}-x_{01})(y_{161}-y_{11})-(z_{02}-z_{01})(y_{11}-y_{01})(x_{161}-x_{11})=0\end{aligned} \qquad \text{(22 to 27)}$$

Equations 1 to 27 define a system of non-linear algebraic equations that must be satisfied for all the PIPs. Grouping the coordinates in a vector $\mathbf{q}$, the equations 1 to 27 are expressed as:

$$\mathbf{F}(\mathbf{q})=\mathbf{0} \qquad (28)$$

This non-linear equations system can be solved using the Newton-Raphson method. This iterative procedure can be written as:

$$\mathbf{F}(\mathbf{q}_i)=\mathbf{F}_{\mathbf{q}}(\mathbf{q}_i)(\mathbf{q}_i-\mathbf{q}_{i+1}) \qquad (29)$$

The evaluation of all the PIPs is a complex problem that requires the computation of all the real solutions of equation (28). This problem may not have a closed form solution due to the complexity of the algebraic equations 1 to 27. However, it may be solved numerically by means of several finite displacement analyses. In the first step, an initial guess is considered for the vector of coordinates $\mathbf{q}$ and the equation (28) is solved using the Newton-Raphson method. Once one PIP is obtained, the next PIP is obtained rotating one actuator 180º, solving for instance the finite displacement analysis with a step of 30º, in order to avoid the convergence to a solution different from the one sought. This procedure is repeated for each actuator until the 64 PIPs are obtained. This problem has been solved in 32 seconds running a MATLAB program on a Pentium 200 MHz processor.

5. Verification of the PIP Condition

In this Section, a procedure to verify that the positions computed in the previous paragraph are PIPs is introduced. Let consider the constant length conditions for the six rods, given by:

$$(x_{161}-x_{11})^2+(y_{161}-y_{11})^2+(z_{161}-z_{11})^2-L_1^2=(x_{11}^{161})^2+(y_{11}^{161})^2+(z_{11}^{161})^2-L_1^2=0 \quad (30)$$

$$(x_{123}-x_{12})^2+(y_{123}-y_{12})^2+(z_{123}-z_{12})^2-L_2^2=(x_{12}^{123})^2+(y_{12}^{123})^2+(z_{12}^{123})^2-L_2^2=0 \quad (31)$$

$$(x_{123}-x_{13})^2+(y_{123}-y_{13})^2+(z_{123}-z_{13})^2-L_3^2=(x_{13}^{123})^2+(y_{13}^{123})^2+(z_{13}^{123})^2-L_3^2=0 \quad (32)$$

$$(x_{145}-x_{14})^2+(y_{145}-y_{14})^2+(z_{145}-z_{14})^2-L_4^2=(x_{14}^{145})^2+(y_{14}^{145})^2+(z_{14}^{145})^2-L_4^2=0 \quad (33)$$

$$(x_{145}-x_{15})^2+(y_{145}-y_{15})^2+(z_{145}-z_{15})^2-L_5^2=(x_{15}^{145})^2+(y_{15}^{145})^2+(z_{15}^{145})^2-L_5^2=0 \quad (34)$$

$$(x_{161}-x_{16})^2+(y_{161}-y_{16})^2+(z_{161}-z_{16})^2-L_6^2=(x_{16}^{161})^2+(y_{16}^{161})^2+(z_{16}^{161})^2-L_6^2=0 \quad (35)$$

Considering the constant length conditions for the edges of the moving platform together with equations 30 to 35 and taking the time derivatives, the following equation is obtained:

$$\begin{bmatrix} x_{12}^{123} & y_{12}^{123} & z_{12}^{123} & 0 & 0 & 0 & 0 & 0 & 0 \\ x_{13}^{123} & y_{13}^{123} & z_{13}^{123} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{14}^{145} & y_{14}^{145} & z_{14}^{145} & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{15}^{145} & y_{15}^{145} & z_{15}^{145} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_{16}^{161} & y_{16}^{161} & z_{16}^{161} \\ 0 & 0 & 0 & 0 & 0 & 0 & x_{11}^{161} & y_{11}^{161} & z_{11}^{161} \\ x_{161}^{123} & y_{161}^{123} & z_{161}^{123} & 0 & 0 & 0 & x_{123}^{161} & y_{123}^{161} & z_{123}^{161} \\ x_{145}^{123} & y_{145}^{123} & z_{145}^{123} & x_{123}^{145} & y_{123}^{145} & z_{123}^{145} & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{161}^{145} & y_{161}^{145} & z_{161}^{145} & x_{145}^{161} & y_{145}^{161} & z_{145}^{161} \end{bmatrix} \begin{Bmatrix} \dot{x}_{123} \\ \dot{y}_{123} \\ \dot{z}_{123} \\ \dot{x}_{145} \\ \dot{y}_{145} \\ \dot{z}_{145} \\ \dot{x}_{161} \\ \dot{y}_{161} \\ \dot{z}_{161} \end{Bmatrix} = \begin{Bmatrix} x_{12}^{123}\dot{x}_{12}+y_{12}^{123}\dot{y}_{12}+z_{12}^{123}\dot{z}_{12} \\ x_{13}^{123}\dot{x}_{13}+y_{13}^{123}\dot{y}_{13}+z_{13}^{123}\dot{z}_{13} \\ x_{14}^{145}\dot{x}_{14}+y_{14}^{145}\dot{y}_{14}+z_{14}^{145}\dot{z}_{14} \\ x_{15}^{145}\dot{x}_{15}+y_{15}^{145}\dot{y}_{15}+z_{15}^{145}\dot{z}_{15} \\ x_{16}^{161}\dot{x}_{16}+y_{16}^{161}\dot{y}_{16}+z_{16}^{161}\dot{z}_{16} \\ x_{11}^{161}\dot{x}_{11}+y_{11}^{161}\dot{y}_{11}+z_{11}^{161}\dot{z}_{11} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (36)$$

The right hand side term in the first row is the dot product between the velocity of the centre of the universal joint of the first rod and a vector pointing in the direction of the rod. It is clear that in a PIP this dot product vanishes since the actuator and the rod are located in the same plane. The same remark can be made for the remaining rows in (36). Therefore, the velocities of the vertices of the moving platform are equal to zero independently of the velocities of the input elements, showing that the positions obtained in the previous Section are PIPs.

6. Conclusions

In this paper, the concept of “positive insensitivity position” that is well known in planar mechanisms like the slider-crank mechanism or the four-bar linkage is generalised to spatial parallel manipulators and demonstrated for the particular case of a 6-RKS like robot. It is shown that this type of robots presents 64 PIPs. In these singular positions, the moving platform can be positioned with a high accuracy because small perturbations in the position of the input elements have nearly no effect on the platform position. Analogously, the velocity of the platform vanishes for any value of the velocities of the input elements. Another advantage of these PIPs is that the torques exerted by the motors are almost zero because the loads applied to the platform are supported by the reaction forces acting on the kinematic joints.

All the “positive” kinematic characteristics mentioned make the 6-RKS parallel manipulator a machine very useful in different industrial applications in which high accuracy in the positioning of the payload is required, like in machine tool applications.

References

Benea, R. Contribution à L’Étude des Robots Pleinement Parallèles de Type 6R-RR-S, Thèse de L’Université de Savoie, Annecy, 1996.

Freudenstein, F. and Kiss, I. S., Type Determination of Skew Four-Bar Mechanisms, Journal of Engineering for Industria, Vol. 91, 1969, pp. 220-224.

García de Jalón, J. and Bayo, E., Kinematics and Dinamics of Multibody Systems, Springer-Verlag, 1993.

Gosselin, M. C. and Angeles, J., Singularity Analysis of Closed-Loop Kinematics Chains, IEEE Transactions on Robotics and Automation, Vol. 6, Nº 3, 1990, pp. 281-290.

Hunt, K. H., Structural Kinematics of In-Parallel-Actuated Robot-Arms, Journal of Mechanisms, Transmissions, and Automation in Desing, Vol. 105, Nº 4, 1983, pp. 705-712.

Merlet, J. P., Parallel Manipulators, Parte 2, Singular Configurations and Grassmann Geometry, Technical report 791, INRIA, Février 1988.

Nombrail, N. Analyse et Commande d’une Famille de Robots Manipulateurs à Structure Parallèle et Redondante, Thèse de L’École Nationale Supérieure de L’Aéronautique et de L’Espace, 1993.

Stewart, D., A Platform With Six Degrees of Freedom, Proc. Inst. Mech. Engrs., Vol. 180, Pt. 1, Nº 15, 1965-66, pp. 371-378.

Takeda, Y., Funabashi, H. and Ichimaru, H., Development of Spatial In-Parallel Actuated Manipulators With Six Degrees of Freedom With High Motion Transmissibility, JSME International Journal, Series C, Vol. 40, Nº 2, 1997, pp. 299-308.

Zlatanov, D., Fenton, R. G. and Benhabib, B., Identification and Classification of the Singular Configurations of Mechanisms, In J. P. Merlet and B. Rabani (eds.), Computational Kinematics, Kluwer Academic Publishers, 1995, pp. 163-172.