KINEMATICS AND DYNAMICS OF A 6-RUS HUNT-TYPE PARALLEL MANIPULATOR BY USING NATURAL COORDINATES

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Abstract In this paper the kinematics, (position, velocities and accelerations) and the dynamics of a 6-RUS parallel manipulator by using the natural coordinates are analysed. One numerical example of kinematics and inverse dynamics is presented.

Keywords: Kinematics, Dynamics, Multibody, Robotics, Parallel Manipulator.

1. Introduction

Many authors have published different methods on the kinematics and dynamics of different types of parallel manipulators. References on the subject can be found in Merlet 2000 and Urls:

-http://www.sop.inria.fr/coprin/equipe/merlet/merlet.html

-http://www.parallemic.org/

The manipulators with rotating actuators compose a special group within the parallel manipulators. Researches on this type of manipulators can be find references Pierrot et al. 1990, Benea 1996, and Zanganeh et al. 1997.

The dynamic equations can be based on different types of generalized coordinates, but most of the authors select Cartesian translations and Euler rotations because they can be easily related to the manipulator degrees of freedom. In this article, the kinematic and dynamic analysis of a 6-RUS parallel manipulator using natural coordinate is presented.

In this case, the natural coordinates are the Cartesian coordinates of the kinematic pairs between the links that compose the manipulator. The advantage of this method resides in the structure of the constraint equations derivatives and the simplicity of the mass matrices.

2. Manipulator topology and notation

The 6-RKS parallel manipulator was introduced by Hunt 1983 and is composed of two triangular platforms one of them fixed to the ground. On the fixed platform there are six rotating actuators (R) located on the edges of the triangle. These actuators are the input elements on which the motors are located. Each actuator is linked to the moving platform through a rod. In each rod, one tip is linked to an actuator through a universal joint (U) and the other tip is connected to the moving platform by means of a spherical joint (S), as depicted in Fig. 1.

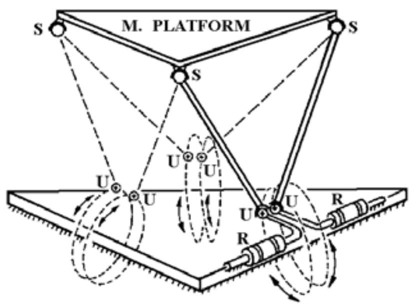

Figure 1. The 6-RUS parallel manipulator introduced by Hunt

The notation used to describe the topology of this parallel manipulator is summarised in the following points and shown in Fig. 2.

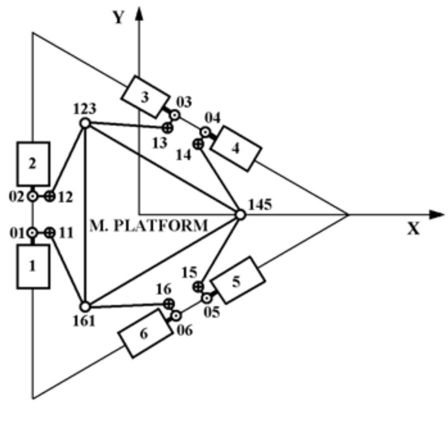

Figure 2. Notation used to describe the parallel manipulator

- *0i:* centre of the rotation axis of the i-th actuator.
- *1i:* centre of the universal joint between the i-th actuator and the i-th rod. The segment defined by points *0i* and *1i* is perpendicular to the actuator axis.
- R_i: length between centres *0i* and *1i*.
- L_i: length of the i-th rod.
- *1ik:* centre of the spherical joint between rods i-th and k-th. These centres coincide with the vertices of the moving platform.

- A_{pq}: length of the edge of the moving platform that links the p-th and q-th rods.

In order to obtain a constant mass matrix for the moving platform, its displacements and rotations must be referred to the position of four non coplanar points. To this end, an auxiliary point (200) is used. It is the vertex of a pyramid whose base is the triangle formed by the vertices of the moving platform.

- A_{2pq}: length of the lateral edge of the pyramid corresponding to p-th and q-th rods.

3. Kinematics

The kinematic analysis comprises the study of position, velocities and accelerations of the manipulator.

3.1 Position

To model the 6-RUS parallel manipulator, a set of *natural coordinates,* García de Jalón et al. 1994, is used. This set of coordinates defines the position of all the elements of the mechanical system with respect to an inertial reference frame. In this case, the generalized coordinates are Cartesian coordinates of different key points as described in section 2. An inertial reference frame is considered on the fixed platform, as shown in Fig. 2, taking the *Z*-axis from the ground towards the moving platform. Giving the lengths R_i, L_i and A_{ij}, the set of constraint equations that define the kinematics of the robot can be written as follows:

- One constant length condition for each actuator:

$$(x_{11}-x_{01})^2+(y_{11}-y_{01})^2+(z_{11}-z_{01})^2-R_1^2=0 \qquad \text{(1 to 6)}$$

- One perpendicular condition for each actuator:

$$(x_{11}-x_{01})(x_{02}-x_{01})+(y_{11}-y_{01})(y_{02}-y_{01})+(z_{11}-z_{01})(z_{02}-z_{01})=0 \qquad \text{(7 to 12)}$$

- One constant length condition for each rod:

$$(x_{161}-x_{11})^2+(y_{161}-y_{11})^2+(z_{161}-z_{11})^2-L_1^2=0 \qquad \text{(13 to 18)}$$

- One constant length condition for each edge of the moving platform:

$$(x_{161}-x_{123})^2+(y_{161}-y_{123})^2+(z_{161}-z_{123})^2-A_{12}^2=0 \qquad \text{(19 to 21)}$$

- One constant length condition for each lateral edge of the pyramid:

$$(x_{200}-x_{161})^2+(y_{200}-y_{161})^2+(z_{200}-z_{161})^2-A_{261}^2=0 \qquad \text{(21 to 24)}$$

In this manipulator is advantageous to introduce 18 linear equations of cranks tips position, of the type:

$$(x_{11}-x_{01})-R_1\cos\theta_1=0 \qquad \text{(25 to 42)}$$

Equations 1 to 42 define a system of non-linear algebraic equations that must be satisfied for all positions. Grouping the coordinates in a vector $\mathbf{q}$, the equations 1 to 42 are expressed as:

$$\mathbf{\Phi}(\mathbf{q}) = \mathbf{0} \tag{43}$$

System (43) is non-linear in the position unknowns. Depending on the number of known positions in vector $\mathbf{q}$, some equations will be directly fulfilled, resulting in a compatible system in the rest of unknowns.

This equation system can be solved using the Newton-Raphson method. The iterative procedure can be written as:

$$\mathbf{\Phi}(\mathbf{q}_i) = \mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}_i)(\mathbf{q}_i - \mathbf{q}_{i+1}) \tag{44}$$

Where $\mathbf{\Phi}_{\mathbf{q}}$ is the jacobian matrix of constraint conditions with respect to the coordinates vector $\mathbf{q}$.

3.2 Velocities

Once the manipulator position is determined, the velocities analysis can be made. The equations system (43) respect to the time is derived obtaining the following system.

$$\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{t}} = \mathbf{0} \tag{45}$$

Where $\dot{\mathbf{q}}$ is the vector of velocities and $\mathbf{\Phi}_{\mathbf{t}}$ is the explicit derivative of the constraint conditions with respect to the time.

System (45) is linear in the velocity unknowns. Depending on the number of known velocities in vector $\dot{\mathbf{q}}$, some equations will be directly fulfilled, resulting in a compatible linear system in the rest of unknowns.

3.3 Accelerations

Once the manipulator velocities are determined, the accelerations analysis can be made. The equations system (45) with respect to the time is derived obtaining the following system.

$$\mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} + \dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} + \dot{\mathbf{\Phi}}_{\mathbf{t}} = \mathbf{0} \tag{46}$$

Where $\dot{\mathbf{\Phi}}_{\mathbf{q}}$ is the derivative of the jacobian matrix $\mathbf{\Phi}_{\mathbf{q}}$ with respect to the time, $\ddot{\mathbf{q}}$ is the accelerations vector and $\dot{\mathbf{\Phi}}_{\mathbf{t}}$ is the second explicit derivative of the constraint conditions with respect to the time.

System (46) is linear in the acceleration unknowns. Depending on the number of known accelerations in vector $\ddot{\mathbf{q}}$, some equations will be directly fulfilled, resulting in a compatible linear system in the rest of unknowns.

4. Dynamics

Applying the method proposed by García de Jalón et al. 1994, the following equations system leads the manipulator dynamics:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{\mathbf{T}}\boldsymbol{\lambda} = \mathbf{Q} \tag{47}$$

where $\mathbf{M}$ is the mass matrix referred to the vector of natural coordinates $\mathbf{q}$, $\boldsymbol{\lambda}$ is the Lagrange multipliers vector and $\mathbf{Q}$ is the vector of applied forces.

In this case, $\mathbf{M}$ contains the dynamic information of the actuator cranks, the rods and the moving platform. The mass matrices of cranks and rods can be referred to two points under certain assumptions and therefore, their mass matrices are of dimension 6x6. The moving platform is represented by four points so as to obtain a 12x12 constant mass matrix.

Taking into a count that all cranks are fixed at one of their ends and the sharing points at the different joint positions, the resulting global mass matrix is of dimension 30x30.

In the more general dynamic analysis, the equations (28) and (29) must be fulfilled at the same time, resulting in:

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_{\mathbf{q}}^{\mathbf{T}} \\ \mathbf{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} \\ -\dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} - \dot{\mathbf{\Phi}}_{\mathbf{t}} \end{bmatrix} \tag{48}$$

System (48) is linear in $\ddot{\mathbf{q}}$ and $\boldsymbol{\lambda}$, but could be reordered for different sets of unknowns. In this case, the unknown $\boldsymbol{\lambda}$'s are directly related to the compression forces in the cranks and rods. Actuator torques can be applied as generalized forces in the corresponding redundant coordinate (see equations 25-42).

In order to solve the inverse dynamic problem, a compatible solution for $\ddot{\mathbf{q}}$ is proposed and system (47) could then be used to determine the unknown actuator torques.

5. Numerical example

In this numerical example, the inverse dynamics problem is solved to determine the actuator torques for a given manipulator kinematics.

The inertia properties of the moving platform are represented in this case by an 80 kg. cylinder of 0.5 m. of diameter and 1.2 m. high which is located on the barycentre of the moving platform.

Fig. 3 shows the topological data of the 6-RUS parallel manipulator. The axes of the actuators mounted on the fix platform form an equilateral triangle of a meter of edge. The actuators are symmetrically mounted with respect to the triangle symmetry axes, being 0,1 m. the distance between the points of the cranks on the axes of the actuators of a same edge of the triangle, that is, the distances between points 01 to 02, 03 to 04 and 05 to 06. The cranks length is 0,1 m. for all actuators. The rods lengths are also all equal to 0,6 m. and the moving platform is an equilateral triangle of 0,5 m. of edge.

The position problem requires six coordinates to be set. In this case, an angle of 30º measured from the interior of the fixed platform is selected for all six crank actuators. The velocity problem is solved by forcing the angular velocity of the actuators to 10, -5, -5, 5, -5 and 10 rad/s. respectively.

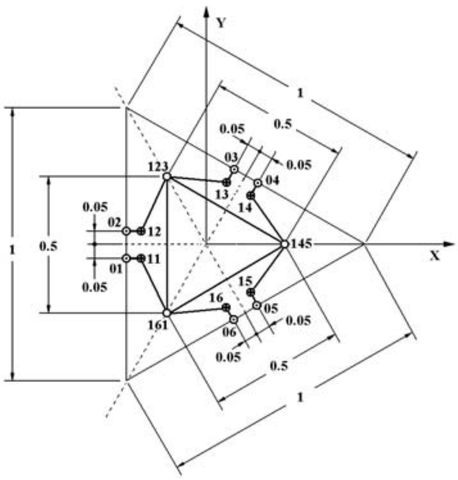

Figure 3. Example of 6-RUS parallel manipulator

5.1 Position

Under the previously mentioned conditions, and using the coordinate system described in Fig. 3 with the Z axis towards the moving platform, the position of all actuators are respectively:

01 = (-0.288675134, -0.05, 0)
02 = (-0.288675134, 0.05, 0)
03 = (0.101036297, 0.275, 0)
04 = (0.187638837, 0.225, 0)
05 = (0.187638837, -0.225, 0)
06 = (0.101036297, -0.275, 0)

Knowing the actuator angles, all crank tip coordinates can be determined by solving the reduced equation system formed by equations (25) to (42).

The position solutions for points 11 to 16 are respectively:

11 =	(-0.202072594,	-0.05,	0.05)
12 =	(-0.202072594,	0.05,	0.05)
13 =	(0.057735027,	0.20,	0.05)
14 =	(0.144337567,	0.15,	0.05)
15 =	(0.144337567,	-0.15,	0.05)
16 =	(0.057735027,	-0.20,	0.05)

Applying the method exposed in section 3.1 the vertices of the moving platform and the vertex of the pyramid (in this case a tetrahedron) are determined, being the following coordinates:

123 =	(-0.144337568,	0.25,	0.612731434)
145 =	(0.2886751346,	0,	0.612731434)
161 =	(-0.144337568,-	0.25,	0.612731434)
200 =	(0,	0,	1.020979724)

5.2 Velocities

Knowing the angular velocities of the actuators, the components of the cranks tips velocities directly determined as:

v11 =	(-0.5,	0,	0.866025)
v12 =	(0.25,	0,	-0.433013)
v13 =	(-0.125,	-0.216506,	-0.433013)
v14 =	(0.125,	0.216506,	0.433013)
v15 =	(-0.125,	0.216506,	-0.433013)
v16 =	(0.25,	-0.433012,	0.866025)

Applying the method exposed in section 3.2, the velocities of the moving platform vertices and the point 200 are determined as follows:

v123 =	(0.441165,	-0.764120,	-0.181050)
v145 =	(0,	-1.528241,	0)
v161 =	(-0.441165,	-0.764120,	0.588414)
v200 =	(0,192033	-0.390562,	0.135788)

5.3 Accelerations

Considering that the angular velocities of the actuators are constant, the components of the cranks tips accelerations are determined directly:

A11 =	(-8.660254,	0,	-5)
A12 =	(-2.165064,	0,	-1.25)
A13 =	(1.082532,	1.875,	-1.25)
A14 =	(1.082532,	1.875,	-1.25)

A15 = (1.082532, -1.875, -1.25)
A16 = (4.330128, -7.5, -5)

Applying the method exposed in section 3.3, the accelerations of the moving platform vertices and the point 200 are determined, being the following components:

A 123 =	(0.610334,	-1.057130,	-2.374374)
A 145 =	(-0.652916,	0,	-7.075207)
A 161 =	(0.972277,	1.684033,	-6.570443)
A 200 =	(4.981187	-3.894854,	-6.397191)

5.4 Dynamics

Applying the method exposed in section 4 for the present inverse dynamics case, the following torques to the motors of 1 to 6 are obtained:

T = (110.82, 91.35, 0, 31.51, -133.37, -145.32)

The traction-compression force on each rod can be deduced directly from Lagrange multipliers as $|F_i| = 2\lambda_i L_i$ resulting:

F = (-1450.24, -1195.46, -1.18, -412.37, 1745.42, 1901.84)

6. Conclusions

In this paper kinematics, (position, velocities and accelerations) and dynamics of a 6-RUS parallel manipulator by using natural coordinates have been analysed and one numerical example of kinematics and inverse dynamics has been presented.

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