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Total and Partial Stationary Configurations for a 6-RUS Hunt-Type Parallel Manipulator

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Abstract: In this paper, the “stationary configurations” (SCs) of a 6-RUS parallel manipulator are analyzed and their use on industrial applications such as positioning mechanisms are discussed. This paper presents the SC concept from a geometric point of view and shows how they can be divided into two categories depending on whether the moving platform remains “practically fixed” under the simultaneous action of all the actuators or only a subset of them. The first category is denoted as “total stationary configurations” (TSCs) while the second group is called “partial stationary configurations” (PSCs). In this work it is calculated the number of TSCs and PSCs the parallel manipulator has. In addition, the spatial surfaces where the platform vertices must be positioned in order to draw the moving platform to either a TSC or a PSC are discussed. A computational method for determining the different TSCs is presented.

Keywords: Stationary configurations, Parallel manipulators

1 Introduction

Some planar and three-dimensional mechanisms show certain particular configurations in which the velocity of the output link of the kinematic chain vanishes independently of the velocity of the input link. As shown in Shigley and Uicker [1], the slider-crank mechanism and the crank-rocker configuration of the four-bar linkage are examples of such systems. In the slider-crank mechanism, two stationary configurations are reached when the crank and the connecting rod are aligned, similarly, in the crank-rocker mechanism, the stationary positions are obtained when the crank and the coupler are aligned.

In this paper, those singular configurations are called “stationary configurations” (SCs). It must be noted that they are positions of the mechanical system in which the position of the output link is set with a very high precision. In those positions small perturbations in the position of the input element have significantly no effect on the position of the output link. It can be said that the output link’s position is “stationary” with regard to the position of the input element.

Singular configurations in which the output element can move while the input element remains fixed can be found. In this paper, these configurations are called “uncertainty configurations” (UCs) and they are not object of this work.

Since the introduction of the Stewart platform [2] (who mentioned the presence of “instability positions”) some authors as Hunt [3], Merlet [4], Gosselin and Angeles [5], Gosselin [6], Tahmasebi and Tsai [7], Zlatanov et al. [8], Basu and Ghosal [9], Karger and Husty [10] and Wohlhart [11] have addressed the problem of SCs and UCs in parallel manipulators with several degrees of freedom.

For parallel manipulators with rotatory actuators, similar to the one object of this work, Pierrot et al. [12] tackled the problem of the singular configurations of the HEXA robot, Takeda et al. [13] determined the regions of the workspace where singular configurations of a parallel manipulator may be present, Benea [14] studied the singular configurations of the parallel manipulator 6-RUS.

Zabalza [15] showed that the usual approach to singular configurations of the majority of researchers is to determine the UCs in order to eliminate them from the actual workspace of the parallel manipulator. Usually no special attention is paid to the existence of SCs or, in the better case, they are treated as if they were UCs. However, SCs may show important benefits in some industrial applications because these configurations are almost “stationary” with regard to the small perturbations that may appear in the actuators position. The benefits of working with SCs are shown in this paper.

2 SCs on 6-RUS Parallel Manipulator

The 6-RUS parallel manipulator introduced by Hunt is depicted in figures 1 and 2. This parallel robot is composed of two triangular platforms, one of them fixed to the ground. On the fixed platform there are six rotating actuators (R) located on the edges of the triangle. These actuators are the input elements on which the motors are located. Each actuator is linked to the moving platform through a rod. In each rod, one tip is linked to the crank of an actuator through an universal joint (U) and the other tip is connected to the moving platform by means of a spherical joint (S), as depicted in figure 1. A couple of rods converge on each spherical joint of the moving platform.

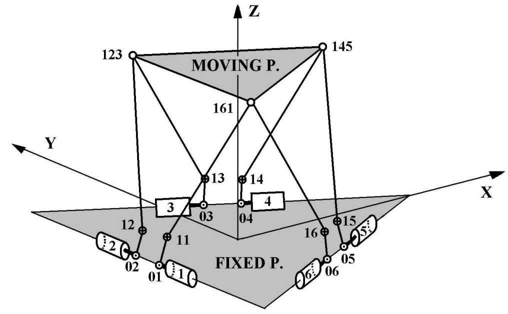

Figure 1 Hunt's 6-RUS parallel manipulator

The notation used to describe the topology of this parallel manipulator is summarized in the following points and shown in figure 2.

- 0i: Center of the rotation axis of the i-th actuator.
- 1i: Center of the universal joint between the i-th actuator and the i-th rod. The segment defined by point 0_i and point 1_i is perpendicular to the actuator axis.
- R_i: Length of the crank between centers 0_i and 1_i.
- L_i: Length of the i-th rod.
- 1ik: Center of the spherical joint between rods i-th and k-th. These centers coincide with the vertices of the moving platform.
- A_{pq}: Length of the edge of the moving platform that links the p-th and q-th rods.

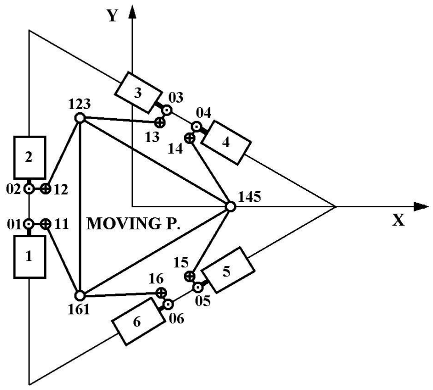

Figure 2 Hunt's 6-RUS parallel manipulator nomenclature

By simple inspection of the kinematic structure of this parallel manipulator, it can be intuitively concluded that “stationary configurations” (SCs) will be present within the manipulator workspace. The different kinematic loops are composed of two slider-crank like mechanisms linked together in the moving platform. By analogy with the planar slider-crank mechanism, it can be expected that SCs will appear in all those positions in which a rod and its corresponding actuator are located in the same plane. A similar condition was introduced by Harrisberger [16] and Freudenstein and Kiss [17] as a condition to determine the limits of the workspace for a spatial four-bar mechanism. This condition was also used by Nombrail [18] and Benea [14] in the characterization of “serial” singularities.

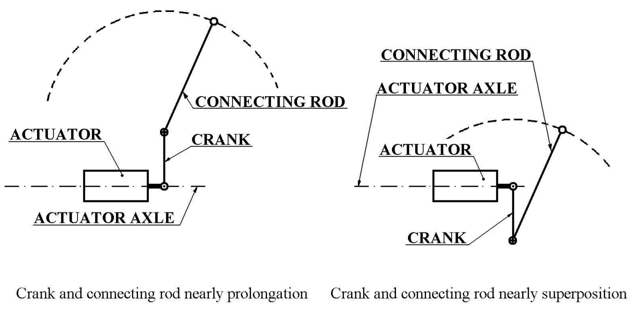

Figure 3 SCs over the fixed platform

Assuming that the rotating actuator may perform full rotations (360º) with respect to the actuator axis, each actuator has two positions in which an SC can be reached (see figure 3). In the first configuration, the actuator crank is located over fixed platform and the rod is nearly aligned with it; in the second configuration, the actuator crank is located under the fixed platform and the rod is nearly superimposed to it. In both cases, the tip of the rod is located over the ground. Similar positions in which the tip of the rod are located under the ground can be described (see figure 4). However, those positions are not considered in this paper because the moving platform would have to go through the fixed platform, which in practice is a non feasible motion.

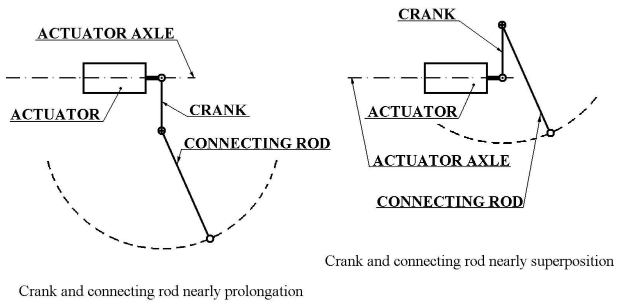

Figure 4 SCs under the fixed platform

In those positions where an SC may appear, the tip of the rod connected to the moving platform may ideally lie on a toroidal surface (Figure 5).

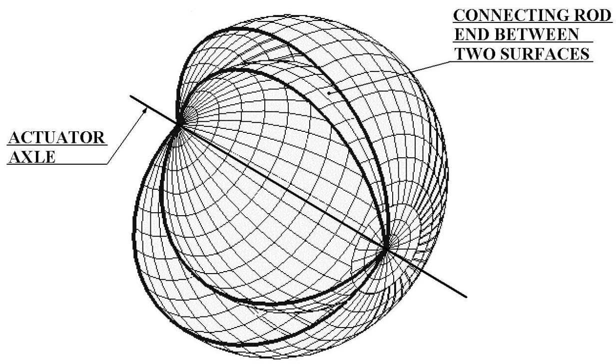

Figure 5 3D view of the toroidal surface

The tips of a single rod are linked to the moving platform and to one of the actuators, respectively. With the condition that the actuator may describe 360º rotations around the actuator axis, for each possible SC the opposite tip of the rod may lie on a circle contained in the plane defined by the actuator crank tip and the actuator axis. Considering all the possible positions of the actuator, it can be easily seen that the tip of the rod may lie on a toroidal surface (Figure 6).

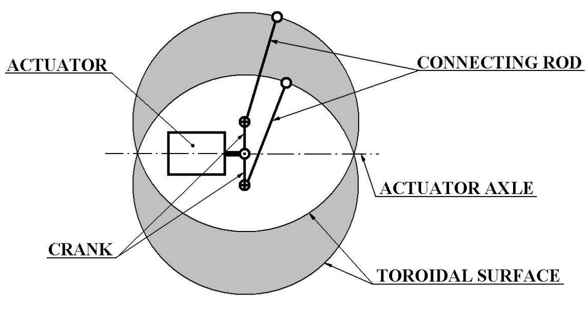

Figure 6 Section of toroidal surface

Only the part of this surface located over the fixed platform is considered (Figure 7).

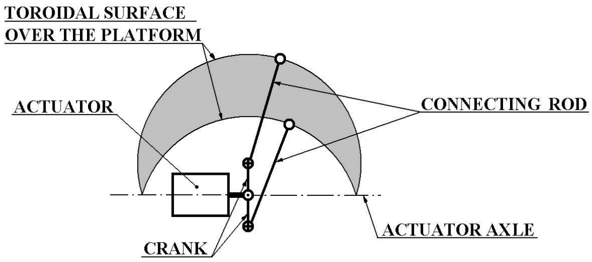

Figure 7 Section of toroidal surface over the fixed platform

Two different rods are joined to each vertex of the moving platform. In order that the moving platform reaches an SC, each rod must lie on the toroidal surfaces described in the previous paragraph. This implies that each vertex of the moving platform must simultaneously be on the toroidal surfaces corresponding to the rods joined in that vertex. In other words, each vertex must be on the intersection of the corresponding toroidal surfaces. The intersection of such surfaces is a 'banana-like' solid, as depicted in figure 8.

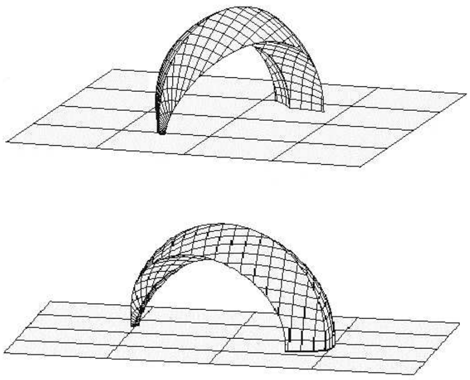

Figure 8 Two view of the banana-like

Figure 14 shows the three 'banana-like' solids corresponding to the three vertices of the 6-RUS parallel manipulator. When the three vertices of the moving platform lie on the surfaces or the edges of those solids, the parallel manipulator will reach a total or partial "stationary configuration".

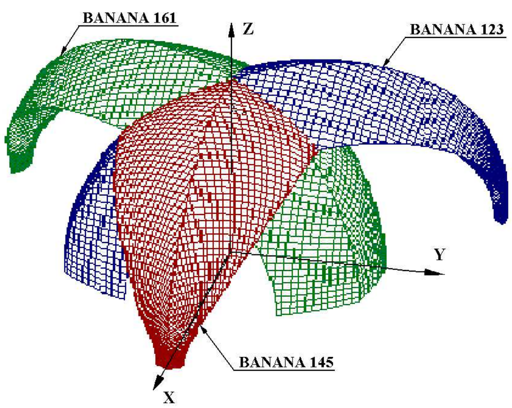

Figure 9 View of 3 banana-like solids

2.1 Total stationary configurations (TSCs)

The 6-RUS parallel manipulator will reach a "total stationary configuration" when the six "legs" reach an stationary configuration. In this situation, small perturbations in the actuators will have no effect in the position of the moving platform. These configurations are denoted as TSCs.

The 6-RUS parallel manipulator is composed of six "legs" or kinematic chains. Each leg is composed of one actuator and a rod connected to the moving platform. Theoretically, a total number of 4^6= 4096 TSCs could be expected. In practice, in each leg, half of these TSCs are not feasible because they imply that the moving platform moves below the platform fixed to the ground. Consequently, only 2^6= 64 TSCs are usually considered. This is also the number of "série" singularities reported by Benea in [14], which directly correspond to the TSCs defined in this paper.

These TSCs are particular positions of the parallel manipulator in the space. In these positions, the vertices of the moving platform are positioned in the intersections of the different surfaces of the 'banana-like' solids that define the regions where singular positions can be found for each robot leg.

2.2 Partial stationary configurations (TSCs)

The 6-RUS parallel manipulator will be in a "partial stationary configuration" when only some of the robot legs reach an stationary configuration. In this particular case, the moving platform will not be affected by small perturbations introduced in those legs that reached the stationary configuration. At the other side, the moving platform will be affected by displacements introduced on the input elements that are not in a stationary configuration.

Partial stationary configurations are identified through the set of input elements or legs that reached a stationary configuration. Following this criteria, the 6-RUS parallel manipulator has 6x2 = 12 partial stationary configurations of type 1 (PSC-1). The concept can be generalized so as to define the partial stationary configurations of type n, which will be denoted PSC-n, and indicates that n input elements have reached a singular position. It can be easily shown that the 6-RUS parallel manipulator has 60 positions of type PSC-2. Among those configurations, 48 correspond to the cases in which the rods located in singular positions do not joint at the same vertex of the moving platform; the remaining 12 positions correspond to those cases in which the rods that are at singular positions join at the same point of the moving platform. In this case, the tips of the rods will be on the edges of the 'banana-like' solids.

Following this reasoning, it can be found that the 6-RUS parallel manipulator shows 120 PSC-3, 240 PSC-4 and 192 PSC-5. In those configurations, the tips of the rods may be either on the edges or at the surfaces of the 'banana-like' solids described in the previous section.

3 Practical Functionality of the SCs

In many practical applications, mechanisms with stationary configurations are used. In particular, planar mechanisms as the slider-crank or the four-bar crank-rocker mechanisms are often used when rotational motions must be converted into longitudinal or oscillatory motions, respectively. In both types of mechanisms the output element shows an alternate motion with limit positions on which the output link stops. These limit positions are an example of stationary configurations

(SCs). SCs show some peculiarities that make them very interesting from a design point of view.

- SC are configurations on which the output link can be positioned with very high accuracy because small perturbations on the position of the input elements do not affect the position of the output link.
- At an SC, the velocity of the output link vanishes thus allowing to define stop positions at the design stage of the mechanical system. Those stop positions can be defined as the limits of the workspace of the mechanical system thus reducing the inertia forces in the limit positions.
- At an SC, the motor effort (torques or forces) that must be applied to balance the forces acting on the mechanism may be zero, at least for a theoretical point of view. In practice, those motor efforts can be dramatically reduced because the applied forces are mainly balanced by the reactions at the kinematic joints.

Analogously to the planar mechanisms mentioned in the previous paragraphs, some parallel manipulators show SCs within their workspace. In particular, the 6-RUS Hunt manipulator studied in this paper shows SCs that can successfully applied in applications like positioning devices. The parallel manipulator can be used as support for the pieces being manufactured. If the parallel manipulator is designed in such a way that the TSCs can be used as machining positions, high accuracy positions can be achieved and small perturbation in the actuators will have practically no effect in the position and orientation of the work piece. The parallel manipulator can be stopped on those positions without the need of equilibrating high inertia forces. In addition, the torques applied by the actuators to keep the robot in equilibrium will be very small since the reaction forces at the kinematic joints will balance the external forces. PSCs have mechanical properties similar to those described for TSCs with the particularity that only those “legs” of the robot that are in a stationary configuration show those properties.

4 Calculating the 64 TSCs for the 6-RUS Manipulator

In this section, a numerical method to determine the 64 TSCs is presented. This method is based on the use of natural coordinates, which are described in detail in García de Jalón and Bayo [19]. The set of natural coordinates is a set of Cartesian coordinates that define the position of all the elements of the mechanical system with respect to the inertial reference frame. This set of Cartesian coordinates is composed of the coordinates of some points and the components of some unit vectors, Those points and unit vectors are typically defined on the kinematic joints of the mechanical system. In this way, the natural coordinates are shared between contiguous bodies so as keeping moderate the total number of unknowns in the kinematic and dynamic analyses.

To model the 6-RUS parallel manipulator studied in this paper, a fixed reference frame is considered on the triangle fixed to the ground. The origin of the reference frame is taken on the center of gravity of the triangle and the axes X, Y and Z are taken as shown in figure 2. Assuming that the positions of the actuators are given and the geometric dimensions of the parallel manipulator (lengths of rods, dimensions of the moving platform, lengths of the actuators, etc.) are known, the constraint equations that define the kinematics of the mechanisms can be written as follows. The notation introduced in section 2 is used. To define the constant length condition for the actuator crank, a set of six quadratic equations, like:

$$(x_{11}-x_{01})^2+(y_{11}-y_{01})^2+(z_{11}-z_{01})^2-R_1^2=0 \quad (1\text{-}6)$$

are introduced.

The points 11 to 16 are located on the tips of the actuator’s cranks and consequently their trajectories must be circles perpendicular to the rotation axis of the corresponding actuator. These conditions are also defined by means of a set of six quadratic constraints, like:

$$\begin{aligned}&(x_{11}-x_{01})\cdot(x_{02}-x_{01})+(y_{11}-y_{01})\cdot(y_{02}-y_{01})+\\&+(z_{11}-z_{01})\cdot(z_{02}-z_{01})=0\end{aligned} \quad (7\text{-}12)$$

A set of six quadratic constraints like:

$$(x_{161}-x_{11})^2+(y_{161}-y_{11})^2+(z_{161}-z_{11})^2-L_1^2=0 \quad (13\text{-}18)$$

gives the constant length conditions for the rods that joint the moving platform with the actuators.

Rigid body conditions for the moving platform are written in the form of three constant distance conditions, as the one indicated below:

$$(x_{161}-x_{123})^2+(y_{161}-y_{123})^2+(z_{161}-z_{123})^2-A_{12}^2=0 \quad (19\text{-}21)$$

Stationary configurations are expressed using cubic constraints in the natural coordinates. As explained in section 2, SCs can be expected in those positions in which a rod and the actuator the rod is linked to are located in the same plane. Mathematically, this condition can be imposed equaling to zero the volume of the pyramid defined by the two natural points that define the actuator axle and the two natural points that define the position of the rod. Six equations like the following must be satisfied simultaneously in a TSCs.

$$\begin{aligned}&(x_{02}-x_{01})\cdot(y_{11}-y_{01})\cdot(z_{161}-z_{11})-\\&(x_{02}-x_{01})\cdot(z_{11}-z_{01})\cdot(y_{161}-y_{11})+\\&(y_{02}-y_{01})\cdot(z_{11}-z_{01})\cdot(x_{161}-x_{11})-\\&(y_{02}-y_{01})\cdot(x_{11}-x_{01})\cdot(z_{161}-z_{11})+\\&(z_{02}-z_{01})\cdot(x_{11}-x_{01})\cdot(y_{161}-y_{11})-\\&(z_{02}-z_{01})\cdot(y_{11}-y_{01})\cdot(x_{161}-x_{11})=0\end{aligned} \quad (22\text{-}27)$$

Equations (1) to (27) is a system of non-linear algebraic equations that must be fulfilled for all TSCs. If the set of natural coordinates is grouped in a vector of unknowns $\boldsymbol{q}$, the system of non-linear equations can be rewritten in the form:

$$\boldsymbol{\Phi}(\boldsymbol{q})=\boldsymbol{0} \quad (28)$$

where $\boldsymbol{q}$ is the vector that includes the 27 natural coordinates that define the position of the mechanical system. The set of equations (28) can be solved using the Newton-Raphson iterative method. The iteration procedure can be written as:

$$\boldsymbol{\Phi}_{\boldsymbol{q}}(\boldsymbol{q}_{\mathrm{i}})\cdot(\boldsymbol{q}_{\mathrm{i}+1}-\boldsymbol{q}_{\mathrm{i}})=-\boldsymbol{\Phi}(\boldsymbol{q}_{\mathrm{i}}) \qquad (29)$$

The evaluation of all the TSCs is a complex problem that requires the computation of all the real solutions of equation (28). This problem may not have a closed form solution due to the complexity of the algebraic equations (1) to (27). However, the problem may be solved numerically by means of several finite displacement analyses. In the first step, an initial guess is considered for the vector of natural coordinates $\boldsymbol{q}$ and the equation (28) is solved using the Newwton-Raphson iterative method. Once one TSC is found, the next TSC is obtained rotating one actuator 180º, solving for instance the finite displacement problem with increments of 30º, in order to avoid convergence to a solution different from the one sought. This procedure is repeated for each actuator until the 64 TSCs are obtained.

With this algorithm a numerical example has been solved and the 64 TSCs has been obtained in 1.48 seconds running a MATLAB program on a Pentium 2 GHz processor.

5 Conclusions

In this paper, singular configuration characteristics of parallel kinematic machines are studied.

(1) Two types of singular configurations are presented: stationary configurations and uncertainty configurations. Both types of singularities have been studied and described by many authors working in the field of parallel manipulators. In this paper, however, stationary configurations are classified as total and partial stationary configurations and described as useful positions in some industrial applications. SCs provide a way to define with a high accuracy the position of the output link of a complex kinematic chain, independently of small perturbations in the position of one or more input elements.

(2) In the particular case of a 6-RUS parallel manipulator, it has been proved that up to 64 total stationary configurations can be achieved. In addition to the high accuracy, other benefits of TSCs are the dramatic reduction of the inertia forces when the robot is reaching that position as well as the small torques that must be applied by the actuators to keep the machine in static equilibrium. Those particularities allow the selection of asynchronous electric actuators providing a significant reduction in the final cost of the machine.

(3) Finally a numerical method for the computation of the TSCs has been introduced. This is a simple and straightforward method based on the use of natural coordinates to describe the mechanical system. An iterative Newton-Raphson procedure is proposed for the solution of the resulting set of non-linear algebraic equations.

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